Локальные оценки в решении уравнения глобального освещения*

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В статье рассмотриваются локальные оценки метода Монте-Карло в решении уравнения глобального освещения. Локальные оценки позволяют непосредственно вычислять яркость в заданной точке в заданном направлении. На их основе может быть построен алгоритм визуализации трехмерных сцен. Также рассматривается принципиальная возможность видо независимой визуализации трехмерной сцены на основе разложения тела яркости по сферическим гармоникам.

Ключевые слова: *Локальная оценка, двойная локальная оценка, глобальное освещение, Монте-Карло*

Usage of Local Estimations at the Solution of Global Illumination Equation*

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In this article, we consider local estimations of the Monte Carlo method for solving the equation of the global illumination. The local estimations allow directly calculating the luminance at a predetermined point in a given direction. Whereby based on it can be built visualization of 3D scenes. Also considering the possibility of a species independent visualization of 3D scene based on the brightness object decomposition in spherical harmonics.

Keywords: *Local Estimation, Double Local Estimation, Global Illumination, Monte Carlo*

Introduction

Visualization of 3D scenes is produced on the basis of solving the global illumination equation, which represents Fredholm integral equation of the second kind [4]

$$L(\mathbf{r},\hat{\mathbf{l}}) = L_0(\mathbf{r},\hat{\mathbf{l}}) + \\ +\frac{1}{\pi}\int L(\mathbf{r},\hat{\mathbf{l}}')\sigma(\mathbf{r};\hat{\mathbf{l}},\hat{\mathbf{l}}')\left|(\hat{\mathbf{N}},\hat{\mathbf{l}}')\right| d\hat{\mathbf{l}}' \qquad (1)$$

where $L(\mathbf{r},\hat{\mathbf{l}})$ is the radiance at the point $\mathbf{r}$ in the direction $\hat{\mathbf{l}}$, is the bidirectional scattering distribution function (reflectance or transmittance), L_0 is the radiance of the direct radiation straight near the sources, $\hat{\mathbf{N}}$ is the normal at the point $\mathbf{r}$ to the surface of the scene.

The global illumination equation (1) does not have the analytical solution, and the numerical simulation methods are used for its solution such as ray tracing, finite element method and photon mapping.

Among the methods used statistical methods for modeling global illumination equation based on Monte Carlo methods had the most development. In this case all the methods used, including the most advanced, such as Metropolis light transport [5] are of direct simulation Monte Carlo, where the determination of the required characteristics (brightness or luminosity) is based on counting the hits or just raytracing.

In this paper, we propose an alternative approach - local estimations of the Monte Carlo method, which are based not on the count hits in a neighborhood of the point under, but on the evaluation of the probability of transition from the trajectory of a Markov chain to the point [3] [6].

Mathematics

From the integral equation for the solid angle, you can go to the well-known integral equation of Fredholm second kind on surfaces

$$L(\mathbf{r},\hat{\mathbf{l}}) = L_0(\mathbf{r},\hat{\mathbf{l}}) + \\ +\frac{1}{\pi}\int\limits_{(\Sigma)} L(\mathbf{r}',\hat{\mathbf{l}}')\sigma(\mathbf{r};\hat{\mathbf{l}}',\hat{\mathbf{l}})F(\mathbf{r}',\mathbf{r})d^2r' \qquad (2)$$

where

$$F(\mathbf{r}',\mathbf{r}) = \frac{\left|(\hat{\mathbf{N}}(\mathbf{r}),\mathbf{r}-\mathbf{r}')(\hat{\mathbf{N}}(\mathbf{r}'),\mathbf{r}-\mathbf{r}')\right|}{(\mathbf{r}-\mathbf{r}')^4}\Theta(\mathbf{r}',\mathbf{r})$$

and

$$\hat{\mathbf{l}}' = \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

And based on it, you can construct an algorithm for its solution by Monte Carlo method. However, wandering along the Σ surfaces of the scene visualization is not a trivial task. Conventional scheme of wandering of the

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Monte Carlo method in the space, which requires the integral to integration over the volume. Integral over the volume

$$d^3r' = |\mathbf{r}-\mathbf{r}'|^2 dr' d\hat{\mathbf{l}}',$$

$$d\hat{\mathbf{l}}' = \frac{\left|(\hat{\mathbf{N}}(\mathbf{r}'), \mathbf{r}-\mathbf{r}')\right|}{(\mathbf{r}-\mathbf{r}')^2} d^2r'. \quad (3)$$

For integration over dr' we will use equivalent transformation with usage δ-function properties

$$\int\limits_{(\Sigma)} L(\mathbf{r}', \hat{\mathbf{l}}')\sigma(\mathbf{r}; \hat{\mathbf{l}}', \hat{\mathbf{l}})F(\mathbf{r}', \mathbf{r})d^2r' \equiv$$

$$\equiv \int\limits_0^\infty \oint L(\mathbf{r}', \hat{\mathbf{l}}')\sigma(\mathbf{r}; \hat{\mathbf{l}}', \hat{\mathbf{l}})\frac{F(\mathbf{r}', \mathbf{r})}{\left|(\hat{\mathbf{N}}(\mathbf{r}'), \hat{\mathbf{l}}')\right|}|\mathbf{r}-\mathbf{r}'|^2 \times$$

$$\times \frac{\left|(\hat{\mathbf{N}}(\mathbf{r}'), \hat{\mathbf{l}}')\right| d^2r'}{|\mathbf{r}-\mathbf{r}'|^2}\delta(\xi_0 - |\mathbf{r}-\mathbf{r}'|)dr' \quad (4)$$

where ξ_0 is a solution of the $\Pi(\mathbf{r}-\xi_0\hat{\mathbf{l}}') = 0$, $\Pi(\mathbf{r}) = 0$ – surface Σ equation. The surface equation can be included directly into (4), because the ratios $\xi_0 - |\mathbf{r}-\mathbf{r}'| = 0$ and $\Pi(\mathbf{r} - |\mathbf{r}-\mathbf{r}'|\,\hat{\mathbf{l}}') = 0$ are equivalent. At that it is important to consider the δ-function properties.

$$\int\limits_a^b \delta(f(x))dx =$$

$$= \frac{1}{\left|\left|\frac{df(x)}{dx}\right|\right|_{x=x_0}} \int\limits_a^b \delta(x-x_0)dx, f(x_0) = 0. \quad (5)$$

Accordingly we will get for global illumination equation

$$L(\mathbf{r}, \hat{\mathbf{l}}) = L_0(\mathbf{r}, \hat{\mathbf{l}}) +$$

$$+\frac{1}{\pi}\int L(\mathbf{r}', \hat{\mathbf{l}}')\sigma(\mathbf{r}; \hat{\mathbf{l}}', \hat{\mathbf{l}})G(\mathbf{r}', \mathbf{r})d^3r', \quad (6)$$

where new geometric factor

$$G(\mathbf{r}', \mathbf{r}) = \frac{\left|(\hat{\mathbf{N}}(\mathbf{r}), \mathbf{r}-\mathbf{r}')\right|}{(\mathbf{r}-\mathbf{r}')^3}\Theta(\mathbf{r}', \mathbf{r})\times$$

$$\times \left|\frac{d\Pi(\mathbf{r}-\xi\hat{\mathbf{l}}')}{d\xi}\right|\Bigg|_{\xi=|\mathbf{r}-\mathbf{r}_0|} \delta\left(\Pi(\mathbf{r}-|\mathbf{r}-\mathbf{r}'|\,\hat{\mathbf{l}}')\right), \quad (7)$$

where $\Pi(\mathbf{r}-|\mathbf{r}-\mathbf{r}_0|\,\hat{\mathbf{l}}') = 0$, $\hat{\mathbf{l}}' = \frac{\mathbf{r}-\mathbf{r}_0}{|\mathbf{r}-\mathbf{r}_0|}$. The solution (6) can be shown as Neumann series

$$L(\mathbf{r}, \hat{\mathbf{l}}) = L_0(\mathbf{r}, \hat{\mathbf{l}}) +$$

$$+\frac{1}{\pi}\int \frac{1}{\pi}\int L_0(\mathbf{r}_1, \hat{\mathbf{l}}_1)\sigma(\mathbf{r}_2; \hat{\mathbf{l}}_1, \hat{\mathbf{l}}_2)G(\mathbf{r}_1, \mathbf{r}_2)d^3r_1\times$$

$$\times\sigma(\mathbf{r}; \hat{\mathbf{l}}_2, \hat{\mathbf{l}})G(\mathbf{r}_2, \mathbf{r})d^3r_1 + \ldots \quad (8)$$

All members of the series - definite integrals, which will be calculated by the Monte Carlo

$$L(\mathbf{r}, \hat{\mathbf{l}}) = L_0(\mathbf{r}, \hat{\mathbf{l}}) +$$

$$+\frac{1}{\pi}\frac{1}{N}\sum_{i=1}^{N}\frac{L_0(\mathbf{r}_{1i}, \hat{\mathbf{l}}_{1i})}{p_1(\mathbf{r}_{1i}, \hat{\mathbf{l}}_{1i})}\frac{\sigma(\mathbf{r}; \hat{\mathbf{l}}_{1i}, \hat{\mathbf{l}})G(\mathbf{r}_1, \mathbf{r})}{p_2(\mathbf{r}_{1i}, \hat{\mathbf{l}}_{1i} \to \mathbf{r}, \hat{\mathbf{l}})} +$$

$$+\frac{1}{\pi^2}\frac{1}{N}\sum_{i=1}^{N}\frac{L_0(\mathbf{r}_{1i}, \hat{\mathbf{l}}_{1i})}{p_1(\mathbf{r}_{1i}, \hat{\mathbf{l}}_{1i})}\frac{\sigma(\mathbf{r}_{2i}; \hat{\mathbf{l}}_{1i}, \hat{\mathbf{l}}_{2i})G(\mathbf{r}_{1i}, \mathbf{r}_{2i})}{p_2(\mathbf{r}_{1i}, \hat{\mathbf{l}}_{1i} \to \mathbf{r}_{2i}, \hat{\mathbf{l}}_{2i})} \times$$

$$\times\frac{\sigma(\mathbf{r}; \hat{\mathbf{l}}_{2i}, \hat{\mathbf{l}})G(\mathbf{r}_{2i}, \mathbf{r})}{p_2(\mathbf{r}_{2i}, \hat{\mathbf{l}}_{2i} \to \mathbf{r}, \hat{\mathbf{l}})} + \ldots \quad (9)$$

Join the sums into one

$$L(\mathbf{r}, \hat{\mathbf{l}}) = L_0(\mathbf{r}, \hat{\mathbf{l}}) +$$

$$+\frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{\pi}\frac{L_0(\mathbf{r}_{1i}, \hat{\mathbf{l}}_{1i})}{p_1(\mathbf{r}_{1i}, \hat{\mathbf{l}}_{1i})}\frac{\sigma(\mathbf{r}; \hat{\mathbf{l}}_{1i}, \hat{\mathbf{l}})G(\mathbf{r}_1, \mathbf{r})}{p_2(\mathbf{r}_{1i}, \hat{\mathbf{l}}_{1i} \to \mathbf{r}, \hat{\mathbf{l}})} +\right.$$

$$+\frac{1}{\pi^2}\frac{L_0(\mathbf{r}_{1i}, \hat{\mathbf{l}}_{1i})}{p_1(\mathbf{r}_{1i}, \hat{\mathbf{l}}_{1i})}\frac{\sigma(\mathbf{r}_{2i}; \hat{\mathbf{l}}_{1i}, \hat{\mathbf{l}}_{2i})G(\mathbf{r}_{1i}, \mathbf{r}_{2i})}{p_2(\mathbf{r}_{1i}, \hat{\mathbf{l}}_{1i} \to \mathbf{r}_{2i}, \hat{\mathbf{l}}_{2i})} \times$$

$$\left.\times\frac{\sigma(\mathbf{r}; \hat{\mathbf{l}}_{2i}, \hat{\mathbf{l}})G(\mathbf{r}_{2i}, \mathbf{r})}{p_2(\mathbf{r}_{2i}, \hat{\mathbf{l}}_{2i} \to \mathbf{r}, \hat{\mathbf{l}})} + \ldots\right). \quad (10)$$

The last expression can be interpreted as a Markov chain wandering ray with the contribution by the kernel

$$k(x_i \to x) = \frac{\sigma(\mathbf{r}; \hat{\mathbf{l}}_i, \hat{\mathbf{l}})G(\mathbf{r}_i, \mathbf{r})}{p_2(x_i \to x)}. \quad (11)$$

Note that the geometrical form factor (7) contains δ-function, which makes it impossible to direct modeling.

Geometric explanation is shown in Figure 1, which shows the inability to calculate the contribution from the node of the Markov chain to the examined point in examined direction. Feature can be eliminated by integrating over the angle that will be equivalent to the calculation of illumination at a given point $\mathbf{r}$. This approach is called the local estimation.

To calculate the brightness directly, you can enter additional intermediate node corresponding to the point of intersection with the element of the scene in the opposite direction $\hat{\mathbf{l}}$ from the point $\mathbf{r}$. This approach is called double local estimate [1]. Figure 2 shows an example of the process for determining the point of visualization.

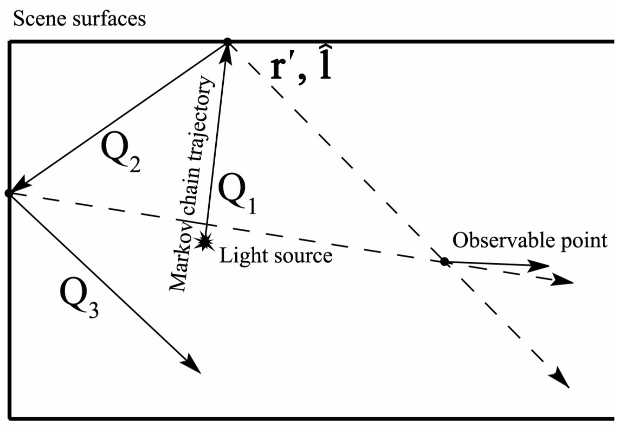

Figure 1: Geometric description of the impossibility of calculating the contribution of brightness from the node of the Markov chain to the studied point in the study area

Implementation and practice

In our work we are working on the implementation of algorithms of local and double local estimations in the .NET Framework environment. Using the managed code is not highly effective in terms of performance, but it allows to focus on the algorithms during the search of optimum.

In the implementation of algorithms in three-dimensional computer graphics directly ray tracing takes significant amount of time. In order to abstract from the problems of optimization of search of intersections through space and focus on our algorithms of local estimations we used the Intel Embree [2] library. High efficiency and simplicity of the interface of this library makes it easy to use it.

Currently in program implemented diffuse rectangular light source, and as a reflection model used mirror, Phong and diffuse reflection models.

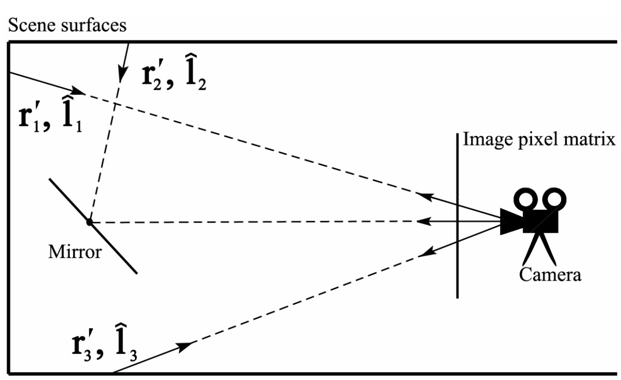

Figure 2: Scheme for determining the points of visualization

Let's consider the basic steps of the visualization of three-dimensional scenes by double local estimation.

In accordance with the general algorithm for calculating the brightness by double local estimation [1], we must determine points and directions (points of visualization), in which we will make the calculation. During visualization of three-dimensional scene we determine these points as points of intersection of the rays that forming the images, issued from the camera. And if the material has a reflection component, then a chain of visualization points is iteratively built.

Figure 2 shows a schematic diagram of determination of visualization points. The brightness of these particular points on specified directions will determine the image. If there are refractive materials in the scene, visualization points are formed similar to the reflection points.

Calculation of high order reflections by double local estimations is a process of multiple constructing a Markov chain of rays from the light source and the calculation of the probability of transition from the anchor point to the point of visualization. Moreover the image of the whole scene can be obtained by only one ray.

Figure 3 shows the visualization of the famous scene Dabrovic Sponza double local estimation by one ray. After calculating the contribution of each point in the image, we are casting a new direction of the ray after reflection and looking for the next point of intersection. From it, we again expect to contribute to all points of the image.

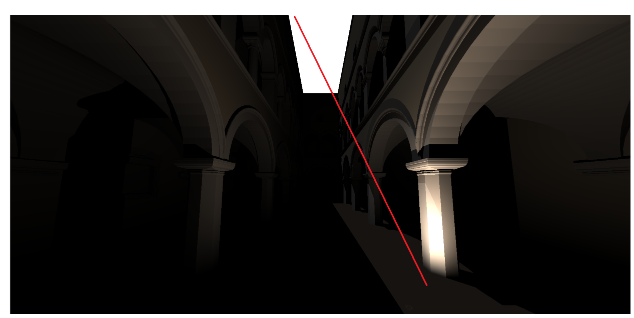

Figure 3: Rendering of the scene, one of multiplicity one ray from the source

The result of accounting of the two multiplicities of the ray reflection is shown in Figure 4.

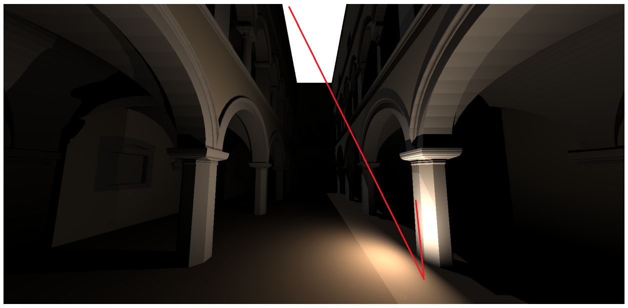

Figure 4: Rendering scene for two multiplicities of one ray from the source

Figure 5 shows an image of one ray with the three multiplicities reflection.

Iteratively, we continue to build a Markov chain and calculate the probability of a transition from a node in the chain calculation points. Chain builds until the weight of the ray above a certain threshold and the ray is not left the scene.

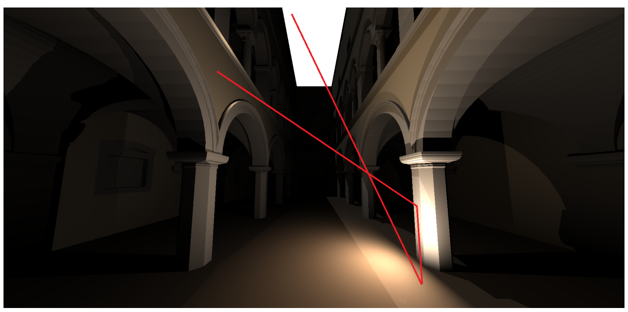

Figure 5: Rendering of the scene in three multiplicities of one ray from the source

Not difficult to see that one ray can really get a full picture of the scene, but it will be biased. Having casted a large number of primary rays, averaging and normalizing the resulting image can be obtained taking into account the multiple reflections of light, shown in Figure 6.

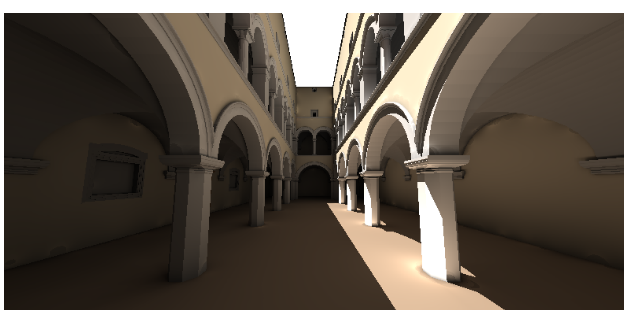

Figure 6: Rendering of the scene by double local estimation

Note that the resulting image contains an explicit artifacts. Not hard to see bursts of brightness near the bottom of the columns, as well as non-uniformity in many areas of the scene containing corners. This is due to the fundamental problem of double local estimations - infinitely dispersion [3]. One approach to solving this problem is to integrate.

At the same time, we note that local estimations devoid of noise, since they use the same rays for all points of the scene, in contrast to the popular method of Metropolis Light Transport.

It should be noted that a similar scheme of rendering the phenomenological approach without strict mathematical basis was obtained for the first time in [8] and called Instant Radiosity. However, the method is based on local estimates entered in the [3] in 1963.

Presentation of the brightness object

Visualizations discussed in the previous section are dependent on the camera position. However, a double local estimation will allow to calculate the brightness at each point for a variety of directions, which opens the possibility of calculation of multiple reflections in the three-dimensional scene without depending on the position of the camera. Moreover, in contrast to the already existing visualization techniques based on the finite element method (radiosity) [7], in this case we do not use diffuse and full reflection model.

This raises the question, how many areas to be used for storage of the brightness at each point. Obviously, the more the better and the more they must be, the more acute the reflection function. At that it is costly to store information. This question requires a separate research, but one solution would be to use the expansion of the angular distribution of brightness in spherical harmonics

$$L(\mathbf{r},\hat{\mathbf{l}}) = \sum_{n=0}^{N}\sum_{m=-n}^{n} C_n^m(\mathbf{r})Y_n^m(\hat{\mathbf{l}}) =$$

$$= \sum_{n=0}^{N}\sum_{m=0}^{n} \left(A_n^m(\mathbf{r})cos\varphi + B_n^m(\mathbf{r})sin\varphi\right) \mathrm{P}_n^m(\hat{\mathbf{l}}\cdot\hat{\mathbf{z}}) \quad (12)$$

where

$$C_n^m(\mathbf{r}) = \oint Y_n^m(\hat{\mathbf{l}})L(\mathbf{r},\hat{\mathbf{l}})d\hat{\mathbf{l}} \quad (13)$$

In this case, in luminance simulation of global illumination equation we can use not a double local estimation but the local estimation directly, the integration is actually taken into account in the deposit. In this case, the expression (13) can be estimated from a random one node and access (9) for computing

$$C_n^m(\mathbf{r}) = \frac{1}{N}\sum_i L_0(\mathbf{r},\hat{\mathbf{l}})Y_n^m(\hat{\mathbf{l}}) +$$

$$+\frac{1}{\pi}\frac{1}{N}\sum_{i=1}^{N}\frac{L_0(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i})}{p_1(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i})}\frac{\sigma(\mathbf{r};\hat{\mathbf{l}}_{1i},\hat{\mathbf{l}})G(\mathbf{r}_1,\mathbf{r})}{p_2(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i}\to\mathbf{r},\hat{\mathbf{l}})}Y_n^m(\hat{\mathbf{l}}) +$$

$$+\frac{1}{\pi^2}\frac{1}{N}\sum_{i=1}^{N}\frac{L_0(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i})}{p_1(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i})}\frac{\sigma(\mathbf{r}_{2i};\hat{\mathbf{l}}_{1i},\hat{\mathbf{l}}_{2i})G(\mathbf{r}_{1i},\mathbf{r}_{2i})}{p_2(\mathbf{r}_{1i},\hat{\mathbf{l}}_{1i}\to\mathbf{r}_{2i},\hat{\mathbf{l}}_{2i})}+ \\ +\frac{\sigma(\mathbf{r};\hat{\mathbf{l}}_{2i},\hat{\mathbf{l}})G(\mathbf{r}_{2i},\mathbf{r})}{p_2(\mathbf{r}_{2i},\hat{\mathbf{l}}_{2i}\to\mathbf{r},\hat{\mathbf{l}})}Y_n^m(\hat{\mathbf{l}})+\ldots \tag{14}$$

The last expression contains integration over the angle that allows you to use directly the local estimate.

Conclusions and future work

Local estimations of the Monte Carlo method allow to calculate directly the brightness at a given point in a given direction. As a result, a double local estimations method can be used to visualize the three-dimensional scene based on multiple reflections.

Based on double local estimations can be built view-independent estimation of the scene, the same method of radiosity. In this case, by contrast, will be used by any model of reflection.

Requires separate consideration of the question of infinite variance and methods of minimizing its impact on the final image.

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