Robust Space/Spatial-Frequency based Filtering of Images in the presence of heavy tailed noise

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Abstract

A robust space/spatial-frequency representation for analysis of two-dimensional signals corrupted by heavy tailed noise is proposed. The L-estimate forms of robust short-time Fourier transform and spectrogram for 2D signals are introduced. They are used to define a non-stationary space-varying filtering procedure in the space/spatial-frequency domain. The efficiency of the proposed procedure is tested on the examples with interferogram images affected by a mixed Gaussian and impulse noise.

Keywords: *robust space/spatial-frequency distributions, L-estimate spectrogram, robust space-varying filtering.*

1. INTRODUCTION

In many real applications, such as communications, signals are often corrupted by the noise. Depending on its statistics, the noise is usually Gaussian, impulse, or the mixture of these two that is generally considered as a heavy tailed noise. The heavy tailed noise is present in images as film grain noise, photoelectronic noise, salt and pepper noise, etc.

Analysis and processing of stationary signals is performed either in the time/space or in the frequency domain. However, for signals that exhibit highly non-stationary characteristics, the joint time-frequency domain should be used [1],[2], or equivalently the space/spatial-frequency domain in the case of two-dimensional signals [3]. Thus, an efficient approach in image filtering could be obtained by using the concept of space-varying filtering based on the space/spatial-frequency representation [4]-[7]. However, the standard time-frequency and space/spatial-frequency representations produce poor results in the presence of heavy tailed noise. This kind of noise requires a special approach and formulation of the robust forms. Therefore, various robust forms of time-frequency distributions have been introduced for one-dimensional signals [8]-[9], for instance, the robust spectrogram and short-time Fourier transform. It has been shown that the L-estimation signal representation provides the best results for the mixture of Gaussian and impulse noises.

This paper represents an extension of robust approaches to the space/spatial-frequency representations used for non-stationary two-dimensional signals (e.g. images). The two-dimensional forms of L-estimate short-time Fourier transform and L-estimate spectrogram are introduced. Based on the robust representations of noisy images, a procedure for robust space-varying filtering is proposed. A suitable filter support function is defined by applying an energy floor to the L-estimate spectrogram. This approach provides successful image filtering, recovering the image quality.

The paper is organized as follows. The robust L-estimate forms of two-dimensional short-time Fourier transform are proposed in Section II. The robust space-varying filtering approach is proposed in Section III. The experimental results are presented in Section IV, while the concluding remarks are given in Section V.

2. TWO-DIMENSIONAL L-ESTIMATE FORMS OF THE SHORT-TIME FOURIER TRANSFORM AND THE SPECTROGRAM

Time-frequency distributions have been used for analysis of non-stationary signals, corrupted by noise. If the noise has a heavy tailed probability density function, the robust distribution forms should be used. In the sequel, the robust space/spatial-frequency distribution for two-dimensional signals is defined.

Consider the two-dimensional noisy signal:

$$x(n_1,n_2)= s(n_1,n_2)+ n(n_1,n_2), \tag{1}$$

where $s(n_1,n_2)$ is a complex-valued two-dimensional signal corrupted with complex-valued noise $v(n_1,n_2)$. We may assume that the noise represents a mixture of Gaussian and impulse noise that usually appears in real applications. In order to provide an efficient space/spatial-frequency analysis, the L-estimate robust short–time Fourier transform (STFT) can be used [10]. Following the concepts introduced for one-dimensional signals, the two-dimensional L-estimate robust STFT can be defined as follows:

$$\begin{aligned} STFT_L(n_1,n_2,k_1,k_2) = \sum_{p=-N/2}^{N/2-1}\sum_{q=-N/2}^{N/2-1} a_p a_q (r_{p,q}(n_1,n_2,k_1,k_2)+ \\ + j\cdot i_{p,q}(n_1,n_2,k_1,k_2)), \end{aligned} \tag{2}$$

where,

$$\begin{aligned} & r_{p,q}(n_1,n_2,k_1,k_2)\in R(n_1,n_2,k_1,k_2), \\ & R(n_1,n_2,k_1,k_2)=\left\{\mathrm{Re}(x_{n,m}), m_1,m_2\in[-N/2,N/2)\right\}, \\ & i_{p.q}(n_1,n_2,k_1,k_2)\in I(n_1,n_2,k_1,k_2), \\ & I(n_1,n_2,k_1,k_2)=\left\{\mathrm{Im}(x_{n,m}), m_1,m_2\in[-N/2,N/2)\right\}, \end{aligned} \tag{3}$$

and

$$x_{n,m} = x(n_1+m_1,n_2+m_2)e^{-j2\pi(m_1k_1+m_2k_2)/N}. \tag{4}$$

The elements: $r_{p,q}(n_1,n_2,k_1,k_2)$ and $i_{p,q}(n_1,n_2,k_1,k_2)$ are sorted in non-decreasing order as:

$$r_{p,q}(n_1,n_2,k_1,k_2) \le r_{p,q+1}(n_1,n_2,k_1,k_2)$$
$$r_{p,q}(n_1,n_2,k_1,k_2) \le r_{p+1,q}(n_1,n_2,k_1,k_2)$$

and

$$i_{p,q}(n_1,n_2,k_1,k_2) \le i_{p,q+1}(n_1,n_2,k_1,k_2)$$
$$i_{p,q}(n_1,n_2,k_1,k_2) \le i_{p+1,q}(n_1,n_2,k_1,k_2).$$

In order to efficiently remove impulses and preserve good resolution, the weighting coefficients a_p and a_q are designed in analogy with coefficients of α-trimmed filter used for image filtering in the space doamin. For one-dimensional case these coefficients are given as [10]:

$$a_p = a_q = \begin{cases} \dfrac{1}{N(1-2\alpha)+4\alpha}, & \text{for } \; p,q \in [(N-2)\alpha, \alpha(2-N)+N-1] \\ 0, & \text{elsewhere,} \end{cases} \tag{5}$$

where N is even, while the parameter α takes values within the range [0,1/2]. In order to clarify the influence of the parameter α value to the noise reduction let us consider the illustration in Figure 1. Namely, after the sorting operation the coefficients corrupted by impulse noise will be located at the beginning and at the end of the sorted sequence. The noise-free coefficients and coefficients corrupted by Gaussian noise will be in the middle part of the sorted sequence. In order to remove the impulse noise, the weighting coefficients a_p and a_q should be zero in the regions where impulse noise exist. Also, it is well known that standard average filter provide good result in the presence of Gaussian noise. Thus, to remove the Gaussian noise the average value of the middle part of the sorted sequence should be calculated.

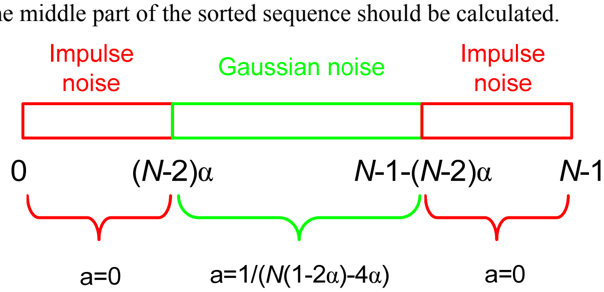

Figure 1. Influence of parameter α to the noise reduction

Higher α provides better reduction of impulse noise, while smaller α improves spectral characteristics. Thus, the value of the parameter α should be carefully chosen. As a special cases, for α=0 and α=1/2 the standard and median distribution forms follow, respectively.

Based on the real and imaginary parts of the L-estimate STFT, the two-dimensional L-estimate spectrogram is defined as:

$$SPEC_L(n_1,n_2,k_1,k_2) = \\ = \mathrm{Re}\{STFT_L(n_1,n_2,k_1,k_2)\}^2 + \mathrm{Im}\{STFT_L(n_1,n_2,k_1,k_2)\}^2, \tag{6}$$

where $STFT_L$ is the L-estimate short-time Fourier transform.

Calculation of the 2D L-estimate STFT

The direct implementation of above equations may result in complex and computationally demanding procedure for calculation of the 2D L-estimate STFT. In order to define more appropriate calculation procedure, let us start with realization of the 2D Fourier transform. It is well known that 2D Fourier transform of a signal $x(n_1,n_2)$ can be calculated as:

$$FFT2(x(n_1,n_2)) = FFT(FFT(x(n_1,n_2))^T)$$

where T denotes transposition operator. Since the STFT represent the windowed version of the Fourier transform, the similar concept can be used for calculation of the 2D L-estimate STFT. The procedure for 2D L-estimate STFT calculation based on the 1D L-estimate STFT is presented in Figure 2. The first step is to multiply image rows with corresponding basis functions, which are the same as in the Fourier transform. In order to distinguish coefficient corrupted by impulse noise and coefficients corrupted by Gaussian noise, the obtained products are sorted. The sorted sequences are further multiplied with weighting coefficients a_q and summed, to obtain the noise free time-frequency representation of image rows. In order to obtain noise free space/spatial-frequency representation the same procedure is applied on the columns of time-frequency representation.

3. ROBUST SPACE/SPATIAL-FREQUENCY BASED FILTERING

The stationary filtering can be used in the case when signal and noise do not overlap in space and frequency domain. Otherwise, the space invariant filtering produces poor results. In these circumstances, the nonstationary space-varying filtering can be used [4]. Namely, the signal and the noise can be separated within the joint space/spatial-frequency domain, which is a basis for an efficient space/spatial-frequency filtering procedure. However, if the present noise is mixed Gaussian and impulse noise, the standard distributions cannot provide good representation [10]. Namely, by using standard distributions (as in [4]), the coefficient affected by impulse noise could be of higher strength than true signal coefficients. It may result in inappropriate filter design, and consequently in poor quality of filtered image. Thus, in the presence of mixed Gaussian and impulse noises, the robust form of space/spatial-frequency representation should be used to derive an optimal nonstationary filter.

In analogy with one-dimensional case, the nonstationary 2D space-varying filtering can be generally written in the form [4]:

$$(Hs)(x,y) = \int_a \int_b h\left(x+\frac{a}{2}, x-\frac{a}{2}, y+\frac{b}{2}, y-\frac{b}{2}\right) \times \\ \times w(a,b)s(x+a,y+b)\,da\,db, \tag{7}$$

where $h(x,y,a,b)$ represents the impulse response of the space-varying 2D filter, while s and w are signal and window function, respectively. The 2D window should be separable and symmetrical. The support function can be defined as follows:

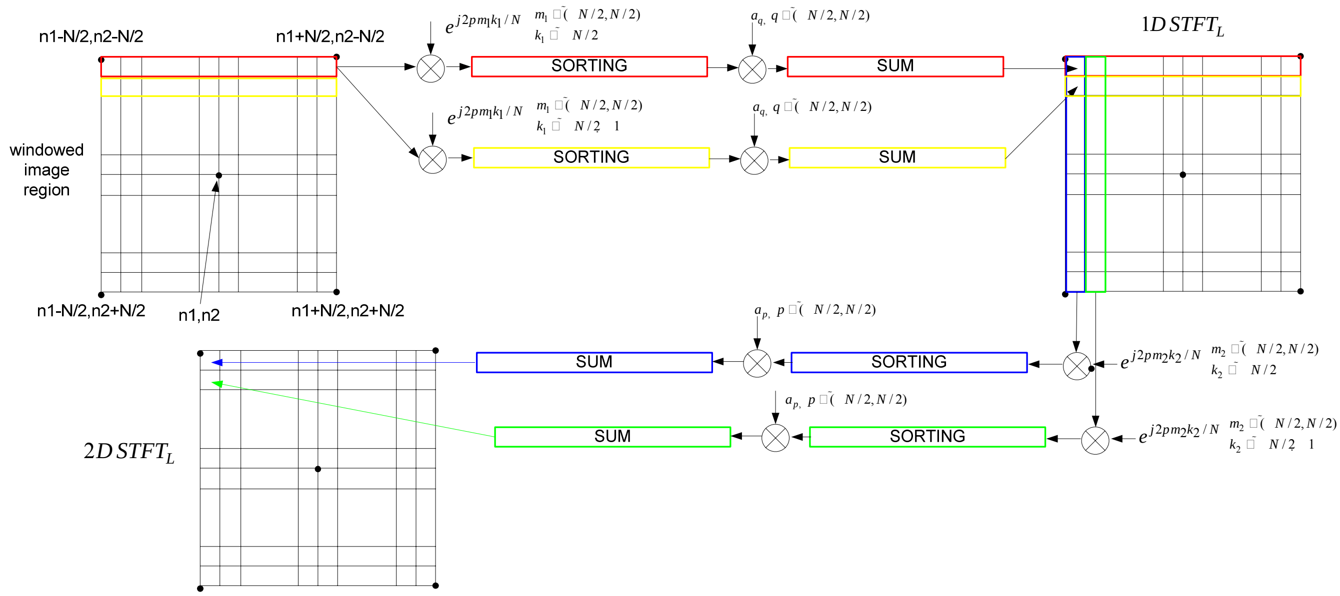

Figure 2. Algorithm for realization of 2D L-estimate STFT based on 1D L-estimate STFT

$$L_H(x,y,\omega_x,\omega_y)=\int_\alpha\int_\beta h(x+\frac{\alpha}{2},x-\frac{\alpha}{2},y+\frac{\beta}{2},y-\frac{\beta}{2})\times \times e^{-j(\alpha\omega_x+\beta\omega_y)}d\alpha d\beta. \quad (8)$$

By using the L-estimate STFT, the robust form of the space-varying filter can be obtained. Namely, based on the Parserval theorem, the output of the L-estimate space-varying filter is defined according to:

$$(H_s^L)(x,y)=\frac{1}{4\pi^2}\int_{\omega_x}\int_{\omega_y} L_H(x,y,\omega_x,\omega_y)STFT_L(x,y,\omega_x,\omega_y)d\omega_x d\omega_y. \quad (9)$$

The support function $L_H(x,y,\omega_x,\omega_y)$ is defined as Weyl symbol mapping to the space/spatial-frequency domain. Assuming that the signal components lie inside the two-dimensional region R_f, while the noise is outside this region, the support function $L_H(x,y,\omega_x,\omega_y)$ is usually defined as:

$$L_H(x,y,\omega_x,\omega_y)=\begin{cases}1 & \text{for} \quad (x,y,\omega_x,\omega_y)\in R_f\\ 0 & \text{for} \quad (x,y,\omega_x,\omega_y)\notin R_f\end{cases}. \quad (10)$$

The discrete form of (10) that is suitable for practical reaizations can be written as follows:

$$(H_s^L)(n_1,n_2)=\sum_{k_1}\sum_{k_2} L_H(n_1,n_2,k_1,k_2)STFT_L(n_1,n_2,k_1,k_2) \quad (11)$$

Therefore, to perform the space-varying filtering we should calculate the L-estimate 2D STFT and determine the support function L_H. Obviously, a precise determination of L_H is related to the precise determination of the region R_f.

Hence, in order to define the support function that will allow efficient noise filtering, we consider the set of points D defined by using the robust L-estimate 2D spectrogram:

$$D=\{(n_1,n_2,k_1,k_2)\,|\,SPEC_L(n_1,n_2,k_1,k_2)>\xi\} \quad (12)$$

where ξ represents the energy floor. Therefore, D contains only the positions of relevant signal components within the space/spatial-frequency representation of noisy signal. The energy floor is defined as a portion of the maximal value of L-estimate 2D spectrogram:

$$\xi=\lambda 10^{\lambda log\left(\max_{k_1,k_2}\{SPEC_L(n_1,n_2,k_1,k_2)\}\right)} \quad (13)$$

Finally, the support function of the nonstationary space-varying filter can be defined in the form:

$$L_H(n_1,n_2,k_1,k_2)=\begin{cases}1 & \text{for} \quad (n_1,n_2,k_1,k_2)\in D\\ 0 & \text{for} \quad (n_1,n_2,k_1,k_2)\notin D\end{cases}.$$

The proposed robust space-varying filtering of images can be summarized through the following steps:

1. Place a center of a two-dimensional window in the pixel on the position (n_1,n_2)
2. Calculate the L-estimate STFT ($STFT_L$) for windowed part of signal

3. Calculate the L-estimate spectrogram $SPEC_L$ based on the $STFT_L$
4. Choose λ and compute the energy floor ξ
5. Determine the support function L_H by using $SPEC_L$ and ξ
6. Calculate the filter output by using L_H and $STFT_L$
7. Repeat the procedure for each image pixel.

A simple block scheme of the filtering procedure is illustrated in Figure 3.

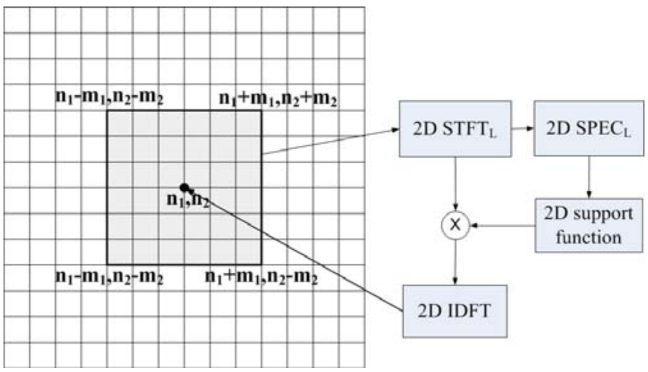

Figure 3. The scheme of the robust space-varying filtering

4. EXPERIMENTAL RESULTS

The proposed approach for the robust space/spatial-frequency based filtering is tested on the numerical example. The analytical image is given in the form:

$$s(n_1,n_2)=\cos\left(20\pi(n_1-0.75)^2+22\pi(n_2-0.75)^2\right)+ \\ +0.5e^{j(-100\cos(\pi n_1/2)-100\cos(\pi n_2/2))}. \quad (14)$$

Note that the considered signal has a form of interferogram image that appears in the optics. The noise is defined as:

$$\nu(n_1,n_2)=0.5\nu_1^3(n_1,n_2)+0.5j\nu_2^3(n_1,n_2) \quad (15)$$

where $\nu_1(n_1,n_2)$ and $\nu_2(n_1,n_2)$ are mutually independent Gaussian noises (zero mean with variance equal to 1).

The original noise-free signal $s(n_1,n_2)$ is shown in Figure 4.a. The signal: $x(n_1,n_2)=s(n_1,n_2)+\nu(n_1,n_2)$ is illustrated in Figure 4.b. Note that the signal $x(n_1,n_2)$ is quite affected by the noise. Firstly, consider the filtering procedures in the spatial domain: the stationary median filtering and filter proposed in [11]. The mask 3x3 is used for booth filters. As shown in Figure 4.c, the median filter produces poor results, and the image quality is even worse than before the filtering operation. The result obtained by filter proposed in [11] (Figure 4.d) are better compared with median filter, but the quality of filtered image is not good especially in the regions with high non-stationarity. Further, we analyzed the procedures in the frequency domain: low pass filtering with cutoff frequency $f=f_{max}/2$ (provide reduction of Gaussian noise) and band-stop filtering with boundaries $f_h=3f_{amx}/4$ and $f_l=f_{amx}/4$ (provide reduction of impulse noise [12]). The results are shown in Figure 4.e and Figure 4.f, respectively. Note that the noise remains present in booth cases, and the quality of filtered image is not satisfactory. The best results are obtained by using the proposed robust space-varying filtering (Figure 4.g). Note that the robust space-varying filtering provides very successful results, providing good quality of filtered image that is close to the quality of original noise-free image.

The proposed robust space-varying filtering is performed by using the L-estimate STFT and spectrogram. The L-estimate forms are calculated by using parameter $\alpha=3/8$. Namely, this value provides satisfying trade-off between noise reduction and distribution concentration. The window size is 64x64. The energy floor is calculated by using $\lambda=0.8$.

5. CONCLUSION

The L-estimation based space-spatial-frequency representation is proposed. Namely, for analysis of two-dimensional signals embedded in heavy tailed noise, the robust forms of multidimensional short-time Fourier transform and spectrogram are introduced. Based on these representations, the concept of non-stationary space-varying filtering is defined. Since it is based on the robust space/spatial-frequency analysis, the proposed procedure provides efficient filtering of a mixed Gaussian and impulse noise. Even for a high amount of noise, the filtered image retains good quality that is close to the original non-noisy image.

6. REFERENCES

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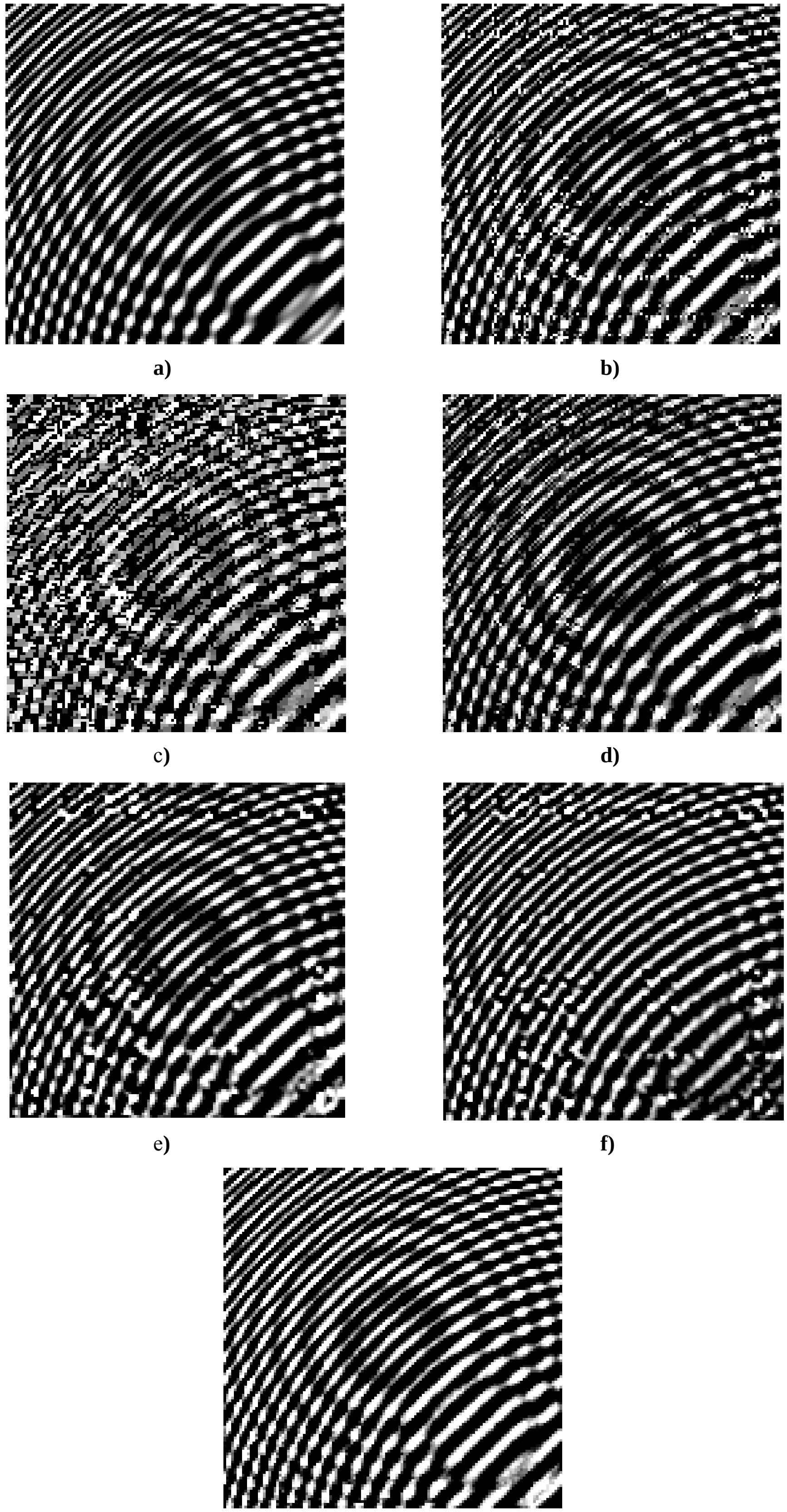

Figure 4: a) original image, b) image corrupted by the mixture of Gaussian and impulse noise, images filtered by: c) median filter, d) filter proposed in [11], e) low pass filter, f) band stop filter, g) space-varying filter based on the L-estimate spectrogram

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