Generalized gamma mixtures for supervised SAR image classification

Vladimir A. Krylov
Faculty of Computational Mathematics and Cybernetics,
Lomonosov Moscow State University,
vkrylov@cs.msu.ru

Josiane Zerubia
EPI Ariana, INRIA/CNRS/UNSA,
INRIA Sophia Antipolis, France,
Josiane.Zerubia@sophia.inria.fr.

Abstract

In this paper we develop a new statistical model for supervised classification of high resolution synthetic aperture radar (SAR) amplitude images. This model is based on the recently proposed generalized gamma distribution (GΓD) for statistics of amplitude SAR images. In order to improve the fit of GΓD when dealing with inherently heterogenous high resolution SAR imagery, we model the statistics of thematic classes as mixtures of GΓD. This enables to consider not homogeneous thematic classes, which is an often requirement in practice. We complete the developed method by proving the identifiability of the developed GΓD finite mixture model and the consistency of the involved parameter estimation scheme (method of log-cumulants) for GΓD, which renders the developed approach mathematically correct. In order to improve the computational performance of the GΓD mixture estimation we suggest the use of an approximative solution of the equations involved, thus, avoiding time-consuming iterative processes. The accuracy of the developed approach is validated on a high resolution TerraSAR-X image and compared to related finite mixture-based SAR classification techniques.

Keywords: *Generalized gamma distribution, finite mixtures, identifiability, method of log-cumulants, consistency, synthetic aperture radar, supervised classification, Markov random field.*

1. INTRODUCTION

The recent progress in sensor and antenna construction enabled the remotely sensed satellite imagery to become widely available and to find its daily applications in fields such as: ecology, meteorology, oceanography, cartography, natural risk management and many others. After optical data, the second most used type of remote sensing imagery is the synthetic aperture radar (SAR) imagery. It has some very important advantages, such as insensibility to Sun-illumination and meteorological conditions [1]. Yet the SAR image processing has some challenges, which are not faced when dealing with optical data. This is due to the speckle noise, which is an inherent phenomenon of the microwave data [1]. In case of the high resolution (HR) SAR, which can be up to 1 m/pixel resolution, the appreciation of details and statistics are heavily affected by speckle.

In this paper we develop an accurate and fast statistical model to deal with one of the basic SAR image processing applications - classification. The purpose of statistical modeling is to get an accurate and concise probability density function (pdf) model that estimates the statistics of a SAR image. In the literature a number of pdf models have been suggested, see in details in [1] and [2], for this purpose, yet in heterogeneous HR case most of them fail. The only solution is to consider mixture-based models and some successful attempts have been reported [2, 3]. In this paper we approach this problem by considering a SAR amplitude pdf as a finite mixture of generalized gamma distributions (GΓD). This is a very flexible family of pdfs [4] that has already found its applications in fields like speech signal processing [5], health and economical applications [6]. GΓD has also been considered for SAR amplitude statistics in a recent work [7]. Here we focus on the mixtures of GΓD, that have not been studied so far mostly due to parameter estimation problems [5].

The paper is organized as follows. In Section 2 we develop the finite GΓD mixture model, state its identifiability (with the outline of the proof in Appendix A), the consistency of the parameter estimation scheme involved (outline of the proof in Appendix B) and suggest an approximative solution of the equations involved (derivation in Appendix C). In Section 3 we merge the statistical pdf model with a Markov random field model to get a contextual SAR classification approach robust with respect to speckle. Section 4 reports the experiments on a high resolution TerraSAR-X image and comparison to related finite mixture-based SAR classification techniques. In Section 5 the conclusions are drawn.

2. GENERALIZED GAMMA MIXTURES

The pdf of the GΓD takes the following form:

$$\mathcal{G}(r|\nu,\kappa,\sigma) = \frac{\nu}{\sigma\Gamma(\kappa)} \left(\frac{r}{\sigma}\right)^{\kappa\nu-1} \exp\left[-\left(\frac{r}{\sigma}\right)^{\nu}\right],\ r \geqslant 0, \quad (1)$$

where ν, κ and σ are positive parameters corresponding to the power, shape and scale, respectively, and $\Gamma(\cdot)$ is the gamma function [8]. The GΓD is a very flexible family of distributions: it includes Gamma, exponential, χ^2, Nakagami, Half-normal, Rayleigh, Maxwell, Weibull, Lévy distributions as special cases and lognormal as an asymptotic case [4, 7].

The use of GΓD to amplitude (intensity) statistics of SAR has been recently suggested by Li et. al [7] and reported good results for HR SAR. Yet the problem of mixed multimodal SAR statistics can not be dealt with even with such a flexible pdf. Thus, in order to take into account this heterogeneity scenario we propose to use a finite mixture model [9] for the distribution of grey levels. We assume an amplitude SAR image $\mathcal{I}$ to be a set $\{r_1,\dots,r_N\}$ of N independent samples drawn from a GΓD mixture pdf with K components:

$$p(r) = \sum_{i=1}^{K} \alpha_i \mathcal{G}(r|\nu_i,\kappa_i,\sigma_i), \quad r \geqslant 0, \quad (2)$$

with mixing proportions $\sum_{i=1}^{K}\alpha_i = 1$, and $\forall i : \alpha_i \in (0,1)$.

The problem of mixture (2) estimation is equivalent to the estimation of $\hat{K}$ and $\{\hat{\alpha}_i,\hat{\nu}_i,\hat{\kappa}_i,\hat{\sigma}_i\}_{i=1}^{\hat{K}}$. In order for this finite mixture estimation problem to be well-posed we show that the following statement holds.

Theorem 1. Finite mixtures of GΓD are identifiable, i.e. if there are two finite GΓD mixtures:

$$H_1 = \sum_{i=1}^{k_1} \alpha_{1i}\mathcal{G}_{1i}, \qquad H_2 = \sum_{i=1}^{k_2} \alpha_{2i}\mathcal{G}_{2i},$$

$\mathcal{G}_{ai} \equiv \mathcal{G}_{aj}$, i.e. $(\nu_{ai},\kappa_{ai},\sigma_{ai}) = (\nu_{aj},\kappa_{aj},\sigma_{aj}) \Leftrightarrow i=j$, for $a = 1,2$, and $H_1 \equiv H_2$, then $k_1 = k_2$ and $\{(\alpha_{1i},\mathcal{G}_{1i})\}_{i=1}^{k_1}$ is a permutation of $\{(\alpha_{2i},\mathcal{G}_{2i})\}_{i=1}^{k_1}$.

The outline of the corresponding proof is given in Appendix A.

The problem of finite mixture estimation is usually too complicated to find a solution directly by a maximum likelihood (ML) approach. Thus, a family of iterative Expectation-Maximization (EM) methods is usually used to numerically find the local maximum of the likelihood function [9]. There exists a considerable variety of EM-method modifications each suitable for some specific problem settings. In our case the optimal approach is presented by a Stochastic EM (SEM) approach [9], which enables to avoid an unfeasible (due to very complex shape of likelihood function [5]) GΓD pdf maximization and to improve the exploratory properties of EM, which is very critical in case of inaccurate (random) initialization. For the SEM algorithm the complete data is represented by a set $\{(r_i, s_i), i = 1, \dots, N\}$, where r_i are the observations (SAR amplitudes) and s_i - the missing labels: given an mixture with K components, a label $s_i \in \{\sigma_1, \sigma_2, \dots, \sigma_K\}$ assigns the i-th observation to one of the K components.

We embed the Method of Log-Cumulants (MoLC) [10] method for parameter estimation in the M-step of SEM scheme. This is done instead of the ML-estimate which come at too high computational price for GΓD [5]. MoLC represents a parametric pdf estimation technique suitable for distributions defined on $[0, +\infty)$, and it has been widely applied in the context of SAR-specific parametric pdf families due to its analytical adequacy for the multiplicative models, like "signal-speckle" system for SAR [2, 3, 7, 10]. MoLC adopts the Mellin transform [10] by analogy to the Laplace transform in the moment generating function. Given a non-negative random variable u, one defines the "second-kind characteristic function" ϕ_u of u as the Mellin transform [10] $\mathcal{M}$ of the pdf of u, i.e.:

$$\phi_u(s) = \mathcal{M}(p_u)(s) = \int_0^{+\infty} p_u(u) u^{s-1} du, \quad s \in \mathbb{C}.$$

The derivatives $\kappa_\nu = [\ln \phi_u]^{(\nu)}(1)$ are the νth order log-cumulants, where $^{(\nu)}$ stands for the νth derivative, $\nu \in \mathbb{N}$. In case of the transform convergence, the following MoLC equations take place [10]:

$$\kappa_1 = E\{\ln u\}, \quad \kappa_j = E\{(\ln u - \kappa_1)^j\},\ j = 2, 3$$

First, we find sample estimates for these log-cumulants [11]:

$$\tilde{\kappa}_1 \approx k_1 = \frac{1}{N} \sum_{i=1}^{N} \ln r_i, \quad \tilde{\kappa}_2 \approx k_2 = \frac{1}{N-1} \sum_{i=1}^{N} [\ln r_i - k_1]^2,$$

$$\tilde{\kappa}_3 \approx k_3 = \frac{N}{(N-1)(N-2)} \sum_{i=1}^{N} [\ln r_i - k_1]^3.$$

Then, we analytically express κ_j as functions of unknown parameters and replacing then κ_j by their sample estimates k_j we finally get the system of MoLC-equations. In case of GΓD it writes [7]:

$$k_1 = \log \sigma + \frac{\Psi(0, \kappa)}{\nu}, \tag{3}$$

$$k_j = \frac{\Psi(j-1, \kappa)}{\nu^j}, \quad j = 2, 3, \dots, \tag{4}$$

where $\Psi(n, x),\ n \in \{0\} \cup \mathbb{N}$, denotes the polygamma function [8].

The following theorem explores the asymptotical properties of the MoLC estimates for GΓD.

Theorem 2. The sequence of MoLC estimates $\{(\hat{\nu}_n, \hat{\kappa}_n, \hat{\sigma}_n)\}_{n=1}^{\infty}$ calculated for GΓD from (3)-(4) and observations $\{r_i\}_{i=1}^{n}$ is consistent, i.e. it converges in probability to the true parameter values $(\nu^*, \kappa^*, \sigma^*)$ as $n \to \infty$.

The outline of the corresponding proof is given in Appendix B.

As demonstrated in Appendix B the system (3)-(4) is invertible, yet its analytical solution is unfeasible. We propose to use an approximative solution of this system that is very simple computationally and it is given by:

$$\begin{cases} \hat{\kappa} = \left[\left(\left[\frac{\alpha}{2}\right]^2 + \left[\frac{\beta}{3}\right]^3 \right)^{1/2} - \frac{\alpha}{2} \right]^{1/3} + \\ \qquad + \left[- \left(\left[\frac{\alpha}{2}\right]^2 + \left[\frac{\beta}{3}\right]^3 \right)^{1/2} - \frac{\alpha}{2} \right]^{1/3} - \frac{c_2}{3c_3}, \\ \hat{\nu} = \left[\frac{\Psi(1, \hat{\kappa})}{k_2} \right]^{1/2} \\ \hat{\sigma} = \exp\left[k_1 - \frac{\Psi(0, \hat{\kappa})}{\hat{\nu}} \right] \end{cases}$$

with

$$c_0 = k_3^2 - 8k_2^3,\ c_1 = 6k_3^2 - 16k_2^3,\ c_2 = 12k_3^2 - 8k_2^3,\ c_3 = 8k_3^2, \tag{5}$$

$$\alpha = \frac{27c_3^2 c_0 - 9c_3 c_2 c_1 + 2c_2^3}{27c_3^3}, \quad \beta = \frac{c_1}{c_3} - \frac{1}{3}\left(\frac{c_2}{c_3}\right)^2. \tag{6}$$

The derivation of these solutions is presented in Appendix C.

Thus, the tth iteration of histogram-based SEM for GΓD mixture estimation goes as follows:

• **E-step:** compute, for each greylevel z and i-th component, the posterior probability estimates corresponding to the current pdf estimates, i.e. $z = 0, \dots, Z-1$:

$$\tau_i^t(z) = \frac{\alpha_i^t \mathcal{G}_i^t(z)}{\sum_{j=1}^{K_t} \alpha_j^t \mathcal{G}_j^t(z)}, \quad i = 1, \dots, K_t,$$

where $\mathcal{G}_i^t(\cdot)$ is the σ_i-conditional pdf estimate on the tth step.

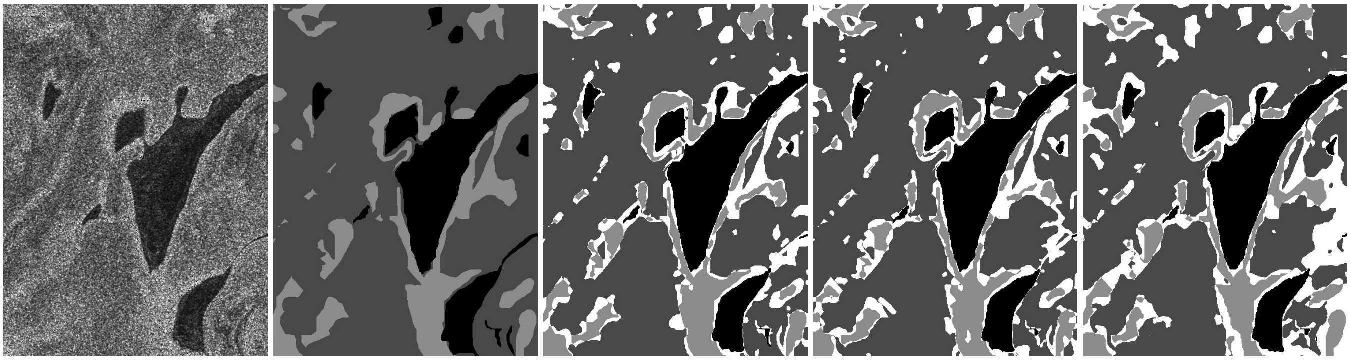
• **S-step:** sample a label for each greylevel z according to the current estimated posterior probability distribution $\{\tau_i^t(z) : i = 1, \dots, K_t\}$, $z = 0, \dots, Z-1$.

• **M-step:** $\forall i$, compute the sample estimates of the log-cumulants (k_{1i}, k_{2i}, k_{3i}) and mixture proportion α_i. Find then the parameter estimates $(\alpha_i^{t+1}, \nu_i^{t+1}, \kappa_i^{t+1}, \sigma_i^{t+1})$ as above.

• **K-step:** $\forall i$: if $\alpha_i^{t+1} < \gamma$, eliminate the i-th component, update K_{t+1}. The choice of threshold γ is not critical, e.g. $\gamma = 0.005$.

3. SUPERVISED SAR CLASSIFICATION WITH GΓD MIXTURES

The problem of classification consists in attributing to each pixel of the considered image $\mathcal{I}$ a label assigning it to one of the M thematic classes. Working in the mainframe of the supervised classification we consider some training pixels to be available for each of the M classes. First, we perform the learning of the statistical properties of each class by estimating a GΓD mixture pdf on its training pixels, thus getting pdfs $p_i(r), i = 1, \dots, M$. It is possible then to get the first classification by picking for each pixel the label with the maximal value p_i, i.e. the ML-classification, however this gives very noisy results. This is especially critical for SAR images, damaged by speckle [1]. Thus, some regularization has to be performed. Here, we suggest the use of a Markov random field (MRF) model which takes into account the local context information [12]. To implement the MRF-energy minimization we run the Modified Metropolis Dynamics (MMD) algorithm, which is a compromise



(a) Initial VV pol SAR image (b) Ground truth (c) DSEM classification (d) GΓD classification (e) Nakagami classification

Figure 1: (a) Initial image, 800x1000 pixels, (b) Ground truth (black - water, dark grey - wet soil, light grey - dry soil), (c) DSEM, (d) GΓD mixture and (e) Nakagami mixture classifications maps (black - water, dark grey - wet soil, light grey - dry soil, white - misclassification).

Table 1: Automatically estimated numbers of mixture components K^* and classification accuracies obtained by considered finite mixture methods: class accuracies and overall accuracies.

Mixture method	Water		Wet soil		Dry soil		Overall
GΓD	88.55%	2	90.31%	3	68.46%	3	**85.95%**
DSEM [2]	88.67%	2	88.86%	2	72.51%	3	**86.19%**
Nakagami [3]	90.24%	2	84.97%	2	64.51%	3	**81.38%**

solution between Iterative Conditional Modes and Simulated Annealing, and, as such, is computationally feasible and provides reasonable results in real classification problems [13]. The employed here MRF and MMD algorithms and their settings are not novel and we refer the reader to an earlier work for more details [14].

4. EXPERIMENTAL RESULTS

The algorithm settings are the following. We initialize $K_0 = 5$ as an overestimate for each class, so that SEM finds K^* by annihilating the redundant components. The number of iterations for SEM is set to $T = 300$. We run the MRF coefficient estimation on the ML-classification and the MMD settings are as in [14].

The experiments were performed on a VV polarization, 6.5 m ground resolution, 2.66-look TerraSAR-X (©Infoterra GmbH, 2008) image acquired over Sanchagang, China. The application was to epidemiological monitoring and the thematic classes were: water, wet soil and dry soil. Figure 1 presents the classification results and comparisons with other mixture-based SAR statistics models, i.e. DSEM model [2] and Nakagami-gamma mixture model based on SEM (as described above), analogical to [3] for amplitudes. Table 1 reports K^* estimates and the obtained accuracies. We observe that DSEM slightly outperforms our model, however this was to be expected as DSEM is based on a mixture of several pdf families, of which GΓD is part. The Nakagami-gamma fit is worse due to a lower flexibility of the parametric pdf model, and we remind that Nakagami (and gamma) pdfs are special cases of GΓD corresponding to $\nu = 2$ ($\nu = 1$) in (1). From the performance point of view, the estimates (averaged over $M = 3$ classes) were obtained in: $T_{\text{DSEM}} = 93s, T_{\text{G}\Gamma\text{D}} = 22s, T_{\text{Nakagami}} = 18s$ on an Intel Core 2 Duo 1.83GHz, 1Gb RAM, WinXP system. This result confirms our GΓD mixture model's flexibility and fast performance.

5. CONCLUSIONS

In this paper we have introduced a novel GΓD mixture estimation approach and demonstrated its efficiency in the application to high resolution SAR image classification. We have proved the identifiability of this type of finite mixtures and demonstrated the consistency of the exploited parameter estimation procedure. In order to improve the performance of the GΓD mixture estimation we suggested the use of an approximative solution of the equations involved, thus, avoiding time-consuming iterative processes. The experimental comparison demonstrated a high potential of this model in the problem of high resolution SAR statistical modeling: it performs almost as well as the most general dictionary-based stochastic expectation maximization (DSEM) problem, and as fast as a very simple Nakagami-gamma mixture-based one. We want to point out that the use of GΓD mixtures and the developed estimation procedure can be interesting to other applications, such as, e.g., speech signal processing, health management and economical applications.

APPENDIX A. OUTLINE OF THEOREM 1 PROOF

To prove the identifiability of finite mixtures of GΓDs we show that the necessary identifiability conditions are fulfilled in the form proposed by Atienza et al. [15]. In order to do so we demonstrate a linear transform $M_{\mathcal{G}}(t) : \mathcal{G}(x) \to \phi_{\mathcal{G}}$, and a point t_0 from the support of $\phi_{\mathcal{G}}$ that enable us to find the total parametric-space ordering of pdfs $\mathcal{G}(t|\nu, \kappa, \sigma)$ that fulfills the following condition:

$$F_1 \prec F_2 \quad \Leftrightarrow \quad \lim_{t \to t_0} \frac{\phi_{F_2}(t)}{\phi_{F_1}(t)} = 0, \tag{7}$$

where F_1, F_2 are two GΓDs.

We define the transform M as follows:

$$M: \ \phi_{\mathcal{G}}(t) = \int_0^\infty t^x \mathcal{G}(x) dx = \sigma^t \, \frac{\Gamma(\kappa + t/\nu)}{\Gamma(\kappa)}$$

and $t_0 = \infty$. We then show that when $t \to t_0$ we have

$$\frac{\phi_{F_2}(t)}{\phi_{F_1}(t)} \sim C \, \exp\{ \left[\frac{1}{\nu_2} - \frac{1}{\nu_1} \right] t \log t + [\kappa_2 - \kappa_1] \log t +$$
$$+ \left[(\log \sigma_2 - \log \sigma_1) - \left(\frac{1}{\nu_2} - \frac{1}{\nu_1} \right) - \left(\frac{\log \sigma_2}{\nu_2} - \frac{\log \sigma_1}{\nu_1} \right) \right] t \}$$

with a constant C, and prove the following ordering satisfies (7):

$$F_1 \prec F_2 \qquad \Leftrightarrow \qquad \begin{array}{l} [\nu_2 > \nu_1] \\ [\nu_2 = \nu_1, \quad \sigma_2 < \sigma_1] \\ [\nu_2 = \nu_1, \quad \sigma_2 = \sigma_1, \quad \kappa_2 < \kappa_1]. \end{array}$$

Thus, finite mixtures of GΓDs are identifiable.

APPENDIX B. OUTLINE OF THEOREM 2 PROOF

To prove the consistency of the sequence of MoLC estimates $\{\hat{\xi}_n\}_{n=1}^{\infty} = \{(\hat{\nu}_n, \hat{\kappa}_n, \hat{\sigma}_n)\}_{n=1}^{\infty}$, i.e. that it converges in probability to the true parameter values $(\nu^*, \kappa^*, \sigma^*)$, we first demonstrate the invertibility of $\Theta : (0, +\infty)^3 \to R^3$ and the continuity of mapping $\hat{\xi}_n = \Theta^{-1}(\hat{k}_n)$, where $\hat{k}_n = (\hat{k}_{1n}, \hat{k}_{2n}, \hat{k}_{3n})$ is the sequence of sample log-cumulants [11]. According to (3), (4),

$$\Theta(\xi) = \left(\log \sigma + \frac{\Psi(0,\kappa)}{\nu}, \quad \frac{\Psi(1,\kappa)}{\nu^2}, \quad \frac{\Psi(2,\kappa)}{\nu^3} \right).$$

This is proved by referring to the implicit function theorem and properties of polygamma functions [8].

Then we take advantage of the fact that the expressions for variances of the sample log-cumulants are well known [11] and show that for any value of $\varepsilon > 0$, if $||\hat{k}_n - k^*|| < \delta(\varepsilon)$, and the central moments $\mathbb{E}\left[(\log X)^j\right]$, $X \sim \mathcal{G}$, till order $j = 6$ exist and are finite, then we get

$$\mathbb{P}\{|\hat{\nu}_n - \nu^*| < \varepsilon, |\hat{\kappa}_n - \kappa^*| < \varepsilon, |\hat{\sigma}_n - \sigma^*| < \varepsilon\} \geqslant 1 - \frac{\sqrt{L(n)}}{\delta(\varepsilon)},$$

where $L(n) = O(n^{-1})$. The derivation of explicit expressions for the necessary log-moments of GΓD demonstrates them to be finite whatever the values of parameters. Thus, we get the required

$$\lim_{n\to\infty} \mathbb{P}\left\{|\hat{\nu}_n - \nu^*| < \varepsilon, |\hat{\kappa}_n - \kappa^*| < \varepsilon, |\hat{\sigma}_n - \sigma^*| < \varepsilon\right\} = 1.$$

APPENDIX C. DERIVATION OF THE CLOSED-FORM ESTIMATOR FOR GΓD PARAMETERS

In order to find a numerical solution of the system (3)-(4) and render the solution computationally as fast as possible we suggest to use an asymptotic decomposition of the polygamma functions [8]:

$$\Psi(1,x) = x^{-1} + 0.5x^{-2} + \bar{o}(x^{-2}),$$
$$\Psi(2,x) = -x^{-2} - x^{-3} + \bar{o}(x^{-3}),$$

with $x \in \mathbb{R}, x \to \infty$.

Isolating ν in (4) for $j = 2, 3$ we search for the solutions of the algebraic equation $c_3\kappa^3 + c_2\kappa^2 + c_1\kappa + c_0 = 0$, with c_i as in (5). By substituting $y = \kappa + c_2/(3c_3)$ we get to the reduced equation $y^3 + \beta y + \alpha = 0$ with α and β as in (6). Now we apply the Cardano's formula and pick the positive solution. Thus, we get the formula for $\hat{\kappa}$, and the solutions for $\hat{\nu}, \hat{\sigma}$ follow from (3)-(4).

We point out that to have a required real value solution $\hat{\kappa}$ using the involved decomposition and Cardano's formula we need the weak following condition fulfilled: $3k_3^2 \leqslant 8k_2^3$. Dealing with real SAR imagery we never stumbled upon a case when this condition fails.

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About the authors

Vladimir A. Krylov is a PhD student at the Dept. of Mathematical Statistics, Faculty of Computational Mathematics and Cybernetics, Lomonosov Moscow State University.
Email: vkrylov@cs.msu.ru

Josiane Zerubia is a senior researcher (Directeur de Recherche), head of the ARIANA research team, INRIA/CNRS/UNSA, INRIA Sophia Antipolis, France.
Email: Josiane.Zerubia@sophia.inria.fr