THE WAY OF AUTOMATION OF GRAPHIC METHOD OF THE SOLUTION OF MATHEMATICAL MODELING PROBLEMS

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Abstract

The offered approach is the further computer development of graphic methods of the solution of mathematical modeling (MM) problems. It will be a question of ways of computer graphic information's organization in the form of special images-models, which reflect geometrical sense of an object that was analytically set. Such way of image graphic representation of complex analytical statements allows to simplify algorithms of problem's solution. Thus the breadth of MM application in the fields of researches allows speaking about prospects of the use of such approach to various classes of applied problems.

Keywords: *Image-model, normal field, recursive algorithm, geometrical modeling, voxel data representation, mathematical programming, optimization problems, gradient method.*

1. INTRODUCTION

The process of visualization of graphic information is unequivocally important, but is far not its unique application. Computer graphic information as the organized image of graphic data can and should be actively applied as analyzing model of investigated object. Illustrativeness of graphic representation in existing graphic means of solution of mathematical modeling problems allows to find result quickly not demanding construction of complex analytical expressions. Thus graphic data should display geometrical properties of investigated function adequately. It is possible to consider such image as a model of function. We organize a process of the computer analysis of mathematical models in a graphic way on the basis of application of graphic images-models.

One of requirements to *computer graphic image-model* should be its *dimensional commensurableness with object-prototype*. In this case only it is possible to speak about adequacy of an *image-model* to the prototype.

The characteristic of change of normal inclination in neighbor points of an object is the invariant, so it's a geometrical property which characterizes a constancy of an object form in various transformations. We allocate this property and model it using graphic images on the basis of changes of color intensity. In such graphic image we organize the local geometrical information, in the form of some set of the images, allowing to automate possibilities of the differential analysis of object-prototype. Such graphic image we will define as *image geometrical model*. As such graphic image can contain not only the information on geometrical properties of a prototype it is possible in general case to speak about some *image-model (M-image)* [1].

In a stage of development of analytical design systems the special role is played by process of formation and the analysis of spatial objects. In this case voxel graphic representation causes special interest for developers of analytical CAD because of orderliness, regularity and index *commensurableness* of elements of an image with the object.

One of the basic differences of *M-images* from a traditional *graphic representation* of projected object is the width of its application in the further automated processes (for example, rendering, optimizing and engineering calculations, etc.). *The M-image* is dimensionally equal with object, contains local geometrical characteristics of an object, and allows to generate new ***M-images.*** Voxel representation is considered as a set of cubic neighborhoods of the points which fill the orthogonal space of geometrical object so, contains possibility of definition of normal's components in these points. The offered way of voxel graphic representation of such information allows to form the *image geometrical information* about analytical object as a *voxel graphic structure*. Under the voxel graphic structure of an image geometrical model here is offered to consider structural organization of M-images as integer scalar fields. Base images of structure display components of a normal field, normalized on the value of a colour palette. Other M-images as structure elements are generated on the basis of base M-images and display differential properties of an object. Base images represent the relational change of components of the normal for the analytically set object (U).

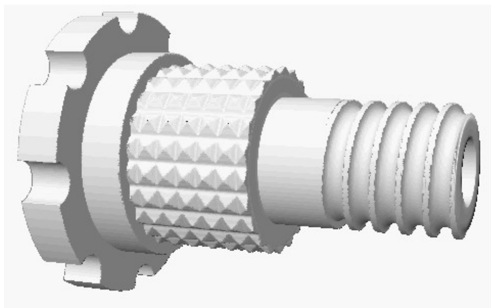

Figure 1: Thread bush, described by a mathematical apparatus of R-functions

The quantity of base images coincides with the dimension of the object (for example, in 3D-case A_u, B_u, C_u are coefficients of tangent plane in the differential neighborhood of a point of the object (U)). At next level images of partial derivatives are automatically determined in plane xOy $\left(\frac{\partial u}{\partial x} = \frac{A_u}{C_u}; \frac{\partial u}{\partial y} = \frac{B_u}{C_u}\right)$. On the base of the images of partial derivatives we can get images of components of the normal fields $(A_{\frac{\partial u}{\partial x}}, B_{\frac{\partial u}{\partial x}}, C_{\frac{\partial u}{\partial x}})$ and $(A_{\frac{\partial u}{\partial y}}, B_{\frac{\partial u}{\partial y}}, C_{\frac{\partial u}{\partial y}})$, enabling acquisition of images of higher derivatives etc. Thus a graphic structure representing geometrical properties of object (U) is generated.

2. THE “RANOK” SYSTEM

The system of analytical designing RANOK (Recursive ANalysis on image Components) is based on the presented principles. It recursively synthesizes the base graphic M-images. Graphic information in base M-images allows rendering the object (Fig. 1)) [2]. Due to this information the RANOK system allows to expand the possibilities in the field of researches of object. A gradient method, allowing to solve optimization problems, is automated. On a Fig. 2 the result of gradient motion for function, describing a «cup» is shown.

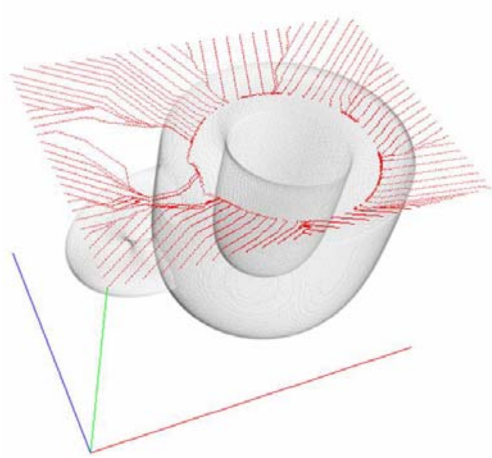

Figure 2: Movement on the gradient descent from the perimeter of a clearance verge

Figure 3 depicts the curve of gradient development w of the considered system enabling one to carry out the long-term planning with allowance for gradual increase of the flows under study. At that, the values of the three flows can be determined at each point of the gradient motion.

3. FORMULATION OF THE OPTIMIZATION PROBLEM OF MATHEMATICAL PROGRAMMING BASED ON THE APPARATUS OF R-FUNCTIONS

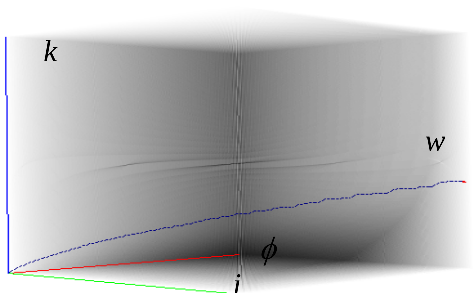

Figure 3: Example for demonstration of a gradient development of the 3-D system.

The equation system with boundary conditions of mathematical programming problem we replace by its logic intersection w

$$w = \bigcap_{i=1}^{n} w_i = w_1 \wedge w_2 \wedge \ldots \wedge w_n .$$

where w_i - equations of boundary conditions.

The objective function F is applied in the geometrical model F_w which is ready to research by gradient method in the RANOK system as follows:

$$\begin{cases} F_w = F + w_0 \cdot (1 + \nabla F) \to \max \\ F_w = F + \left| w_0 \cdot (1 + \nabla F) \right| \to \min \end{cases},$$

where $w_0 = w - |w|$ – is the zeroed domain of permissible plans.

Illustrations of the Examples of Solution of Problems of Nonlinear Programming in the RANOK System are presented in the proceedings [3].

4. CONCLUSION

Computer technologies of graphic methods of solution of tasks is just begins to emerge, but even now it is possible to state that it offers a tool for dealing with a wide range of computer-assisted mathematical applications One of the methods of using the vector fields represented in the raster and voxel images for solution of the gradient problems allows one to reconsider application of the field theory in the automation systems. This approach is characterized by the reduction of the density of the field under consideration to a representation as vivid as possible. The listed possibilities of the system do not describe the entire class of problems solved on the regular voxel structures of the cognitive M-images. Application of R-operations enables one to describe piecewise the domain of permissible solutions and objective functions of any complexity, and the cognitive model images used in the context of the RANOK system would enable one to work with spaces of any dimensionality.

5. REFERENCES

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