Smooth Picture Transformation

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Abstract

A problem of picture transformation is applicable in morphing, photogrammetry and some other scopes. The task we are deal with is to set one-to-one correspondence between all points of two pictures, given few pairs of such points. These pairs usually pointed out by a user.

For some applications it is important to construct smooth and exact picture transformation, that keeps the start point correspondence. Moreover, it is desired to have simple formula for the converse transformation.

The purpose of the paper is to suggest a way of such transformation. Methods used here are not completely new, while the composition of them is. We use triangulation of the set of start points, spline functions to construct smooth transformation and partial affine transformation to have the converse map.

Keywords: *Triangulation, Spline.*

1. INTRODUCTION

Let us consider the following task. For two images there are some pairs of points, a point on the left picture and the other on the right one. It is necessary to construct picture transformation from the left picture to the right one, such that the left point of each match goes to the right point.

It is also requested that the transformation is a) exact, that is, each point from the start matches is going exactly to its image continuous, b) smooth (continuity of the derivatives), c) one-to-one and d) convertible (the converse map should be easily computed). To construct the transformation means to have an algorithm of each point image calculation.

According to the first requirement images of some points are given. The second condition of continuity says that an image of each point should be calculated basing mostly on the nearest given point matches. It leads us to the idea of splitting the whole area to some parts and defining the transformation for each part separately.

For the partition we suggest to accept the well-known principle of Delaunay triangulation. To expand the transformation out of the convex hull of the point matches, we add some points «on the infinity» and include them into the triangulation. As the simple transformation linear interpolation inside the triangles can be used, see the fig.1 below.

Such transformation is not smooth, it has discontinuities of the first derivative on the triangle edges. To solve the problem we use a modification of spline quadric polynomial functions, defined on each triangle. These functions are smooth on the edges and provide exact correspondence between the given matches.

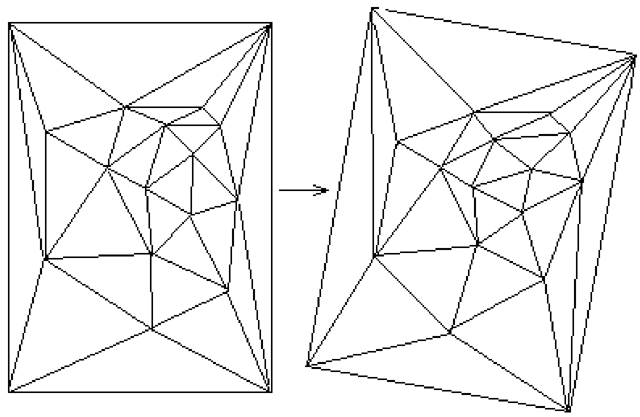

Figure 1: Triangulation

Yet, as most things are, spline function is not perfect and has its own disadvantages. The main of them is non-convertibility, that is, the converse function cannot be easily calculated.

For this matter we insert one more stage - a so-called «regular matrix and base points», which is both a format for data storage and a way of transformation. The idea is easy, we just add regular mesh of points and fix the transformation on the regular mesh and on the original matches. Then the transformation is used as linear interpolation in the triangulation of the regular mesh nodes and base points.

2. TRIANGULATION

To construct the triangles we use a modification of standard divide-and-conquer Delaunay triangulation algorithm. The modification is based on special sorting of points. All the points are sorted by X and divided on two (nearly) equal parts. Then each part is sorted by Y and divided on two parts and so on, see figure 2.

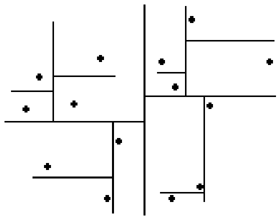

Figure 2: Point partition

The division is finished after each part consists of one point only, as it is shown on the picture. For each point double connected edge list consists of one edge with connections to itself. Then the standard divide-and-conquer procedure works (see [1]).

Such point partition in comparison with the common X-sorted point partition provides absence of long triangles, which is important in the case of huge point sets. Long and narrow triangles generate a problem of insufficient accuracy of calculations.

After Delaunay triangulation on the left point set is constructed, one more problem arises. On the right part correspondent edge links may not form a triangulation, as it is shown on the figure 3 below.

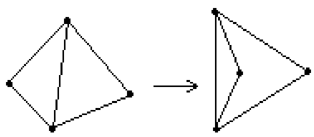

Figure 3: Problems of triangulation

This problem is solved by the well-known flip-flop principle: we change the middle edge on the left picture to the another possible edge. We apply this flip-flop procedure to each pair of adjacent triangles. It gives us proper triangulation on both pictures, if the solution exists.

3. SPLINE FUNCTIONS

These functions work for a closed polygon D, splitted into a union of triangles T_i, such that the intersection T_i and T_j for different i and j either empty or consists of a common edge or a common vertex. We restrict our interest to the class of polynomial functions of degree less or equal to four.

Let us T be a triangle and f is a polynomial of degree d. We consider a homogeneous coordinate system {u, v, w} connected to T be the following way: 1) edges of T has equations u = 0, v = 0 and w = 0; 2) u + v + w = 1. It is easily checked that linear functions u, v, and w are uniquely defined by such equations and triangle T.

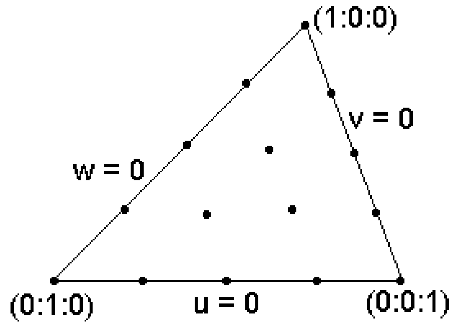

Figure 4: Homogeneous coordinates and base points

The function f can be written as a homogeneous polynomial from the variables u, v and w:

(1)

Such functions will be referred to as Besie polynomial of degree d on T. Let us fix the spatial points P = (u, v, w, z), where (u : v : w) = (a : b : g), $z = z_{abg}$. Then all the meanings of the function f from (1) lies inside the convex hull of the points Pabg.

Projections of the points Pabg on T for the case d = 4 are shown on the figure 4 below. Points Pabg will be called base points of the polynomial (1). Base point (a:b:g) is called a neighbor of a point (r:s:t) if the vector (a:b:g) - (r:s:t) has coordinates 1, -1 and 0 in any order.

Now let us two triangles T_1 and T_2 have a common edge and two Besie polynomial f_1 and f_2 are defined in T_1 and T_2. Suppose we have a general function f, defined in the union of T_1 and T_2 such that the restriction of f on T_i coincides with the function f_i for i = 1,2. We want to know, when f belongs to the function of the class C^1 (functions with continuous derivation). The following two conditions are sufficient:

(i) Base points of T_1 and T_2 on the common edge of these triangles coincide. We denote them as Q and R.

(ii) Any four points P, Q, R and S such that P is a base point of T_1 and is a neighbor for Q and R, while S is a base point of T_2 and is a neighbor for Q and R, lie in one plain in space (are complainer).

The polynomial f is defined by the following base points: one for each vertex, three for each edge and three for each face of the triangulation. Base points, that are in the middle of the edge (point Q on the figure 5), we denote as middle edge points. Points, placed inside of the face and nearest to the vertex, will be referred to as joint points of the vertex (points P and S for the vertex X on the picture).

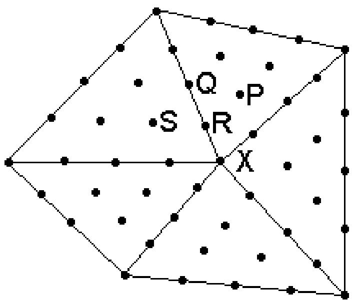

Figure 5: Joint and neighboring base points

A system of linear equations, correspondent to the conditions (i), (ii) in the case d = 4 is block organized. It is also easy to find a set of independent parameters, defining its general solution.

Base points, neighboring to the vertex S – such as point R on the picture – must lie in one plain and the plain is tangential to the surface f in the vertex. Middle edge points we define arbitrary, they are free variables of the system. If in the vertex S values of the function f and its X and Y-derivatives are given, base point S and its neighbors are defined uniquely.

Setting of one of the points joint to S defines other joint points because of the condition (ii). Thus, if S is an inner vertex, we have got an equation, connecting meanings of the derivatives in S and Besie coefficients for one of the base points, joint to S. We suppose here that the middle edge points and vertex base point are fixed.

We will call a vertex even, if the number of the edge from the vertex is even, and odd otherwise. Then the equation for an even vertex does not contain Besie coefficients and connects only derivative values in the vertex. For an odd vertex we have converse situation: Besie coefficient for a base point, joint to S, is defined by the values of the derivatives in the point S.

Therefore, a set of an independent parameters, defining a C^1-function f of degree 4, is the following:

1) Middle base point for each edge
2) Base point for each vertex
3) Values of derivatives and one of the joint base points for each vertex of the convex hull of the triangulation
4) A value of one of the derivatives and one of the joint base points for each even inner vertex
5) Values of derivatives for each odd inner vertex

Total number of free parameters is

r + 3 * vi + 4 * vl,

where r is a number of edges, vi is a number of inner vertices, vl –s a number of outer vertices.

Suppose we have values of polynomial f for each triangulation vertex. How shall we choose free parameters for smooth function? We suggest the following choice. First, a normal vector to the surface in each vertex is appreciated as an average meaning of the normal vectors to all faces containing the vertex.

The middle edge base points are chosen such that the restriction of f on the given edge is a polynomial of the degree 2. Then, for odd and outer vertices we take values of the derivatives in accordance with the normal vectors calculated earlier. For even vertices on the straight line, correspondent to the possible values of the gradient vector (X and Y derivatives for an even vertex are connected by a linear equation), a point nearest to the appreciated normal vector.

Then, for even and outer vertices we choose a joint base point to the given vertex such that the sum of the squares of all distances from the joint points to the plains, passing through the base points for the vertices of the correspondent triangles, is minimal.

4. PARTIAL AFFINE TRANSFORMATION

For a number of applications it is necessary to have not only smooth transformation from one picture to another, but also a converse transformation. For spline functions, described above, converse function is not easily calculated. Moreover, if we have a composition of two or more of such functions, calculation of the converse function is too computation expensive.

To construct a convertible function we suggest representation of spline function as a partial affine transformation. More exactly, we split a picture into a number of triangles, in the vertices of which we calculate the transformation according to the rules described in the previous chapter. The images of the vertices are kept in a special file, that permits us to calculate an image of any point by the following way: a) find a triangle, containing the given point b) compute an image of the point with the help of usual linear interpolation.

To make a partition of the picture into triangles we use regular net of points and base points set by a user. The triangles we define by the following natural way. If a cell of a regular net of points does not contain a base point, it generates two triangles – a rectangular, broken by a diagonal. The direction of a diagonal is fixed. If a cell contains one or more base points, we construct Delaunay triangulation on four vertices of the cell and on the base points inside it.

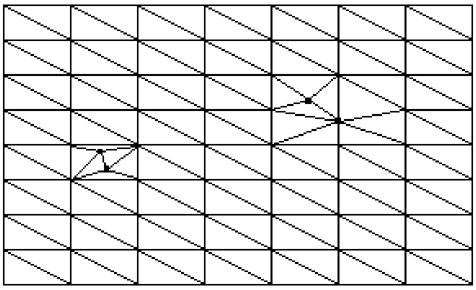

Figure 6: Regular matrix and base points

If one of the base points lies on the bounder – or nearly on the bounder - of a cell of regular matrix, it may cause problems, if on the right picture it goes on the other side of the bounder. To avoid the problem, we insert Delaunay triangulation on the rectangle, which consists of some cells of the regular net.

5. CONCLUSION AND RESULTS

Picture transformation, described in the paper, gives us smooth and convertible function, which can be used in a number of applications. We apply it in Russian photogrammetry system «Talka», while the next example shows another application.

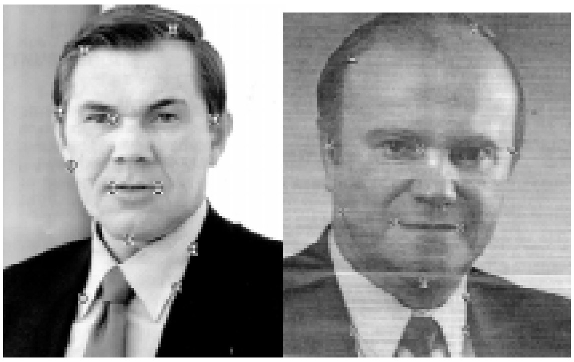

Figure 7: Two man faces with matched points

Figure 7 shows two man faces and some matched points, set by a user. We calculate an affine transformation of one of the pictures to the another such that the square of distances between the matched points is minimal. Then, an average meaning of the points places were detected (that is, for each pair of points we take the middle point).

Thus, we have the same position of the points on an “empty sheet of paper”. Then, the described above transformation were applied from the left picture to this empty sheet and the same for the right one. On the figure 8 we demonstrate left and right pictures, transformed to the same point position.

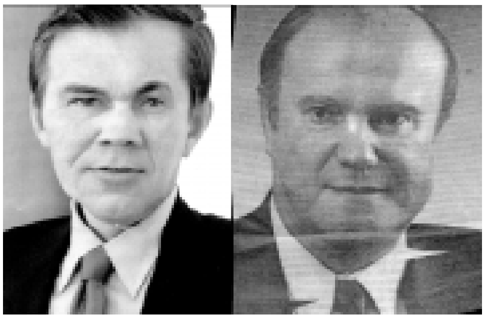

Figure 8: Left and right transformed pictures

In particular, the pictures are nearly the same in geometry, which is illustrated on the figure 9. Here you see a picture, which is a combination of the transformed pictures (from the figure 8). Left part is taken from the left picture, right part from the right one. The seem was set by a user.

Do not wonder to the color adjustment of the pictures. It is a result of a special procedure, which will be described in a separate paper in the next future.

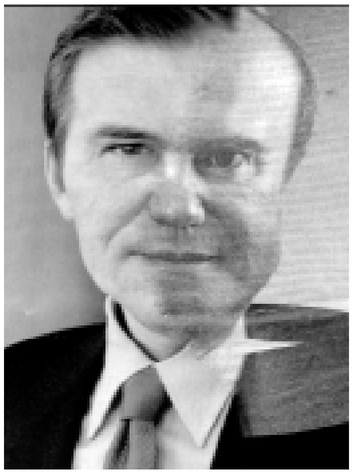

Figure 9: Combined picture

6. REFERENCES

[1] Shamos, Preparata “Computation Geometry”,

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